This equation describes how the temperature gradient is influenced by the opacity, density, luminosity, and radius.

These derivations provide a deeper understanding of the physical processes governing stellar structure and evolution.

### A.6 Luminosity Gradient

The luminosity gradient describes how the luminosity changes with radius within a star.

\*\*Derivation:\*\*

1. \*\*Energy Conservation:\*\*

The rate of energy generation within a shell of radius \( r \) and thickness \( dr \) is:

\[

dL = \epsilon \rho 4\pi r^2 dr

\]

- \( \epsilon \) is the energy generation rate per unit mass (W/kg).

- \( \rho \) is the density (kg/m³).

- \( 4\pi r^2 \) is the surface area of the shell.

- \( dr \) is the thickness of the shell.

2. \*\*Luminosity Gradient:\*\*

Dividing both sides by \( dr \):

\[

\frac{dL}{dr} = \epsilon \rho 4\pi r^2

\]

This equation shows how the luminosity changes with radius due to energy generation within the star.

### A.7 Equation of State for Degenerate Matter

In the cores of white dwarfs and neutron stars, matter is in a degenerate state, where the pressure is dominated by quantum mechanical effects rather than thermal pressure.

\*\*Derivation:\*\*

1. \*\*Electron Degeneracy Pressure:\*\*

For non-relativistic degenerate electrons, the pressure is given by:

\[

P\_e = \frac{(3\pi^2)^{2/3} \hbar^2}{5m\_e} \left( \frac{\rho}{\mu\_e m\_H} \right)^{5/3}

\]

- \( \hbar \) is the reduced Planck constant.

- \( m\_e \) is the electron mass.

- \( \rho \) is the density.

- \( \mu\_e \) is the mean molecular weight per electron.

- \( m\_H \) is the mass of a hydrogen atom.

2. \*\*Neutron Degeneracy Pressure:\*\*

For non-relativistic degenerate neutrons, the pressure is given by:

\[

P\_n = \frac{(3\pi^2)^{2/3} \hbar^2}{5m\_n} \left( \frac{\rho}{m\_n} \right)^{5/3}

\]

- \( m\_n \) is the neutron mass.

These equations describe the pressure in degenerate matter, which is crucial for understanding the structure of white dwarfs and neutron stars.

### A.8 Opacity and Radiative Transfer

Opacity (\( \kappa \)) is a measure of how transparent a material is to radiation. It plays a crucial role in radiative transfer within stars.

\*\*Derivation:\*\*

1. \*\*Rosseland Mean Opacity:\*\*

The Rosseland mean opacity is an average opacity weighted by the temperature gradient:

\[

\frac{1}{\kappa\_R} = \frac{\int\_0^\infty \frac{1}{\kappa\_\nu} \frac{\partial B\_\nu}{\partial T} d\nu}{\int\_0^\infty \frac{\partial B\_\nu}{\partial T} d\nu}

\]

- \( \kappa\_\nu \) is the monochromatic opacity.

- \( B\_\nu \) is the Planck function.

2. \*\*Radiative Transfer Equation:\*\*

The radiative transfer equation in the diffusion approximation is:

\[

\frac{dT}{dr} = -\frac{3 \kappa \rho L}{16 \pi a\_{\text{radiation}} c T^3 r^2}

\]